NAG Toolbox for MATLAB

f08kb

1 Purpose

f08kb computes the singular value decomposition (SVD) of a real m by n matrix A, optionally computing the left and/or right singular vectors.

2 Syntax

```
[a, s, u, vt, info] = f08kb(jobu, jobvt, a, 'm', m, 'n', n)
```

3 Description

The SVD is written as

$$A = U\Sigma V^{\mathrm{T}},$$

where Σ is an m by n matrix which is zero except for its $\min(m,n)$ diagonal elements, U is an m by m orthogonal matrix, and V is an n by n orthogonal matrix. The diagonal elements of Σ are the singular values of A; they are real and nonnegative, and are returned in descending order. The first $\min(m,n)$ columns of U and V are the left and right singular vectors of A.

Note that the function returns V^{T}, not V.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobu – string**

Specifies options for computing all or part of the matrix U.

jobu = 'A'

All m columns of U are returned in array **u**.

jobu = 'S'

The first $\min(m,n)$ columns of U (the left singular vectors) are returned in the array **u**.

jobu = 'O'

The first $\min(m,n)$ columns of U (the left singular vectors) are overwritten on the array **a**.

jobu = 'N'

No columns of U (no left singular vectors) are computed.

Constraint: **jobu** = 'A', 'S', 'O' or 'N'.

2: **jobvt – string**

Specifies options for computing all or part of the matrix V^{T}.

jobvt = 'A'

All n rows of V^{T} are returned in the array **vt**.

jobvt = 'S'

The first $\min(m,n)$ rows of V^{T} (the right singular vectors) are returned in the array **vt**.

jobvt = 'O'

The first $\min(m,n)$ rows of V^{T} (the right singular vectors) are overwritten on the array **a**.

jobvt = 'N'

No rows of V^{T} (no right singular vectors) are computed.

jobvt and **jobu** cannot both be 'O'.

Constraint: **jobvt** = 'A', 'S', 'O' or 'N'.

3: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m** – **int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – **int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldu, ldvt, work, lwork

5.4 Output Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

If **jobu** = 'O', **a** is overwritten with the first $\min(m,n)$ columns of U (the left singular vectors, stored column-wise).

If **jobvt** = 'O', **a** is overwritten with the first $\min(m,n)$ rows of V^{T} (the right singular vectors, stored row-wise).

If **jobu** $\neq$ 'O' and **jobvt** $\neq$ 'O', the contents of **a** are destroyed.

2: **s**$(*)$ **– double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The singular values of A, sorted so that $\mathbf{s}(i) \geq \mathbf{s}(i+1)$.

3: **u**(**ldu**,$*$) **– double array**

The first dimension, **ldu**, of the array **u** must satisfy

if **jobu** = 'S' or 'A', **ldu** $\geq \max(1, \mathbf{m})$;
ldu ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, ucol)$, where $ucol$ is the number of columns of U requested

If **jobu** = 'A', **u** contains the m by m orthogonal matrix U.

If **jobu** = 'S', **u** contains the first $\min(m, n)$ columns of U (the left singular vectors, stored column-wise).

If **jobu** = 'N' or 'O', **u** is not referenced.

4: **vt**(**ldvt**,$*$) **– double array**

The first dimension, **ldvt**, of the array **vt** must satisfy

if **jobvt** = 'A', **ldvt** $\geq \max(1, \mathbf{n})$;
if **jobvt** = 'S', **ldvt** $\geq \max(1, \min(\mathbf{m}, \mathbf{n}))$;
ldvt ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobvt** = 'A', **vt** contains the n by n orthogonal matrix V^{T}.

If **jobvt** = 'S', **vt** contains the first $\min(m, n)$ rows of V^{T} (the right singular vectors, stored row-wise).

If **jobvt** = 'N' or 'O', **vt** is not referenced.

5: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobu**, 2: **jobvt**, 3: **m**, 4: **n**, 5: **a**, 6: **lda**, 7: **s**, 8: **u**, 9: **ldu**, 10: **vt**, 11: **ldvt**, 12: **work**, 13: **lwork**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If f08kb did not converge, **info** specifies how many superdiagonals of an intermediate bidiagonal form did not converge to zero. See the description of **work** above for details.

7 Accuracy

The computed singular value decomposition is nearly the exact singular value decomposition for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. In addition, the computed singular vectors are nearly orthogonal to working precision. See Section 4.9 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is approximately proportional to mn^2 when $m > n$ and m^2n otherwise.

The singular values are returned in descending order.

The complex analogue of this function is f08kp.

9 Example

```
jobu = 'Overwrite A by U';
jobvt = 'Singular vectors (V)';
a = [2.27, -1.54, 1.15, -1.94;
     0.28, -1.67, 0.9399999999999999, -0.78;
     -0.48, -3.09, 0.99, -0.21;
     1.07, 1.22, 0.79, 0.63;
     -2.35, 2.93, -1.45, 2.3;
     0.62, -7.39, 1.03, -2.57];
[aOut, s, u, vt, info] = f08kb(jobu, jobvt, a)
```

```
aOut =
   -0.2774   -0.6003   -0.1277    0.1323
   -0.2020   -0.0301    0.2805    0.7034
   -0.2918    0.3348    0.6453    0.1906
    0.0938   -0.3699    0.6781   -0.5399
    0.4213    0.5266    0.0413   -0.0575
   -0.7816    0.3353   -0.1645   -0.3957
s =
    9.9966
    3.6831
    1.3569
    0.5000
u =
     0
vt =
   -0.1921    0.8794   -0.2140    0.3795
   -0.8030   -0.3926   -0.2980    0.3351
    0.0041   -0.0752    0.7827    0.6178
   -0.5642    0.2587    0.5027   -0.6017
info =
           0
```